Signal Distortion: Measurement Methods and Their Uncertainties

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Abstract

Measurement of distortion is a common test for which there are several largely recognized methods. This paper deals with the uncertainties of these methods and the pitfalls of the evaluation process.

The focus of this paper will be on the methods used in the audio frequency range, mainly for vibration and acoustics. Specifically this deals with the distortion-meter method and the analyzer method (using either FFT analyzer, set of 1/n octave band filters, or selective voltmeter). The analysis will emphasize the key points for the uncertainty evaluation for each method and the difficulties encountered when digital instruments are used.

It is shown that sometimes the analog, old-fashioned methods, are the most accurate and easiest to characterize and further, that we need a common effort to define the measurement errors for the FFT analyzers.

1 Introduction

The signal distortion is a test that is performed frequently, either as a separate parameter or to control the test conditions. A distorted signal contains in its spectrum, besides the fundamental frequency, infinity of additional signals called harmonics, having frequencies that are multiple of the frequency of the fundamental.

The signal distortion, D (or harmonic distortion or total harmonic distortion, THD), represents the sum of harmonics measured RMS, as a percentage of the fundamental tone amplitude. This definition is implemented by the following relation:

$$D(\%) = \sqrt{\frac{\sum_{i=2}^{\infty} A_i^2}{A_1^2}} \cdot 100\ (\%) \qquad (1)$$

where A_1 is the amplitude of the fundamental of the signal, sometimes called the first harmonic, A_i , i = 2, …, n, … are the amplitudes of the harmonics present in the spectrum of the distorted signal.
Often a replacement relation is used:

$$D(\%) = \sqrt{\frac{\sum_{i=2}^{n} A_i^2}{\sum_{i=1}^{n} A_i^2}} \cdot 100\ (\%) \qquad (2)$$

The difference in the measured value between these relations is small enough to justify their interchanged use.

There are various ways to test signal distortion. Some methods lead, in time, to dedicated instruments such as a distortion meter. Other use standard instruments, like signal analyzers. Independent of the degree of automation of the measurements, the difficulties and errors in the measurements remain the same.

2 Distortion meter method

The RMS of the distorted signal is measured and taken as reference (100%) in a first measurement step. In a second step, the fundamental of the signal is rejected with a tunable, notch filter (with attenuation k) and the RMS of the remaining signal residuum is measured, usually directly in percents.

This method is based on the definition through equation (2). Only, in this way in both steps of the test, additional spurious signals are measured (like noise, N, and hum, H) which can seriously affect the measurement as these may be comparable with the magnitude of harmonics. The measurement relation is then:

$$D_m(\%) = \sqrt{\frac{kA_1^2 + \sum_{i=2}^{n} A_i^2 + N^2 + H^2}{\sum_{i=1}^{n} A_i^2 + N^2 + H^2}} \cdot 100\ (\%) \qquad (3)$$

Partial elimination of these interfering signals using filters is not possible for any measured signal. In addition, as shown in Equation (3) due to limited attenuation of the rejection filter, the remnant of the fundamental, $k \cdot A_1$, is also added as a spurious signal.

The uncertainty sources are due: a) to the accuracy of measuring each of the harmonic components present in equation (3), b) to the limited number of harmonics measured because of the limited frequency band of the measuring instrument, c) to the presence of the spurious signals and, d) to the remnant part of the fundamental. We will not discuss here other sources like the environment influence or the short term instability of the test instruments.

a) The uncertainties in the harmonics' amplitudes are evaluated based on the accuracy of the test instrument, which includes the linearity and the flatness of its frequency response, as shown in Figure 1.

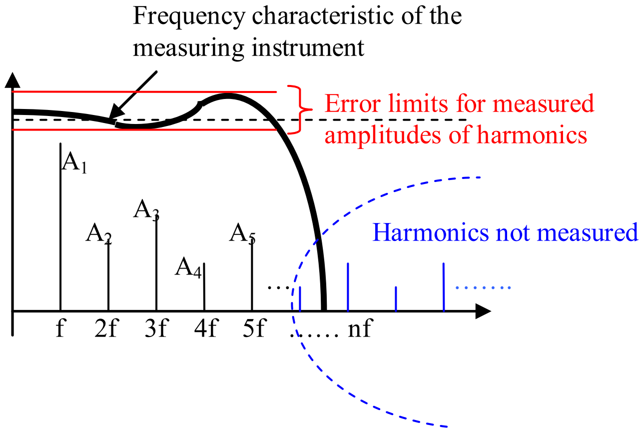

Figure 1. Influence of the measuring instrument frequency characteristic on the uncertainty of measured harmonics amplitudes.

b) The uncertainty due to the amplitudes of the harmonics that should be but are not included in calculations depends on the signal frequency and the frequency range of the distortion meter. In most cases, if ten harmonics are covered then this component is negligible.

c) The level of the spurious hum and noise affecting the measurement can be separately measured in an adequate test arrangement without the signal. Their amplitude can also be calculated from the previously measured signal-to-noise ratio. The variable part of the spurious signals may appear as a type A component. There is still a remaining component of spurious signal, from the steady RMS noise level and the hum.

d) Uncertainty due to the incomplete rejection of fundamental can be estimated based on the filter rejection ratio and the amplitude of the measured signal. In addition, the rejection of fundamental may be influenced by the method and accuracy of tuning the filter on the fundamental frequency: if not perfectly tuned then the filter rejection is not fully used and this uncertainty component is higher. This component may be visible in repeated tests, as part of type A uncertainty.

The calculations for the sensitivity coefficients for the uncertainty components are rather long. Their expressions, in relative values, are given by the following relations:

$$c_{1r} = \frac{\sum_{2}^{n} A_i^2}{A_1^2} \cdot \frac{1}{D^2} = 1 \qquad (4)$$

where c_{1r} is the sensitivity coefficient for the relative uncertainty u_{1r} accounting for the accuracy and stability of the fundamental A_1.

$$c_{ir} = -\frac{A_i^2}{A_1^2} \cdot \frac{1}{D^2} = -\frac{A_i^2}{\sum_{2}^{n} A_i^2} \qquad (5)$$

where c_{ir} is the sensitivity coefficient for the relative uncertainty component u_{ir} accounting for the accuracy and stability of the amplitude of harmonic i, A_i.

The relative uncertainty due to spurious signals influence (noise, hum, remnants of fundamental), expressed as:

$$u_{Nr} = \frac{\sqrt{N^2 + H^2 + k^2 A_1^2}}{\sqrt{\sum_2^n A_i^2}}, \tag{6}$$

is multiplied by:

$$c_{Nr} = \frac{\sum_2^n A_i^2}{A_1^2} \frac{1}{D^2} = 1 \tag{7}$$

The composed relative uncertainty for distortion measurement is calculated using the general relation for composing uncertainties when no correlation is present:

$$u_c^2 = c_{1r}^2 \cdot u_{1r}^2 + \sum_2^n c_{ir}^2 \cdot u_{ir}^2 + c_{Nr}^2 u_{Nr}^2 + s_r^2 = u_{1r}^2 + \frac{1}{D^4} \sum_{i=2}^n \left(\frac{A_i^2}{A_1^2} u_{ir} \right)^2 + u_{Nr}^2 + s_r^2 \tag{8}$$

where s_r is the relative repeatability uncertainty component.

One can see that the influence of the uncertainty components is variable with the value of the distortion and the magnitude of the various harmonics.

These relations apply for the following methods as well. The actual values of the components will be different from one method to another.

3 Analyzer method

This method is based on separately measuring the fundamental and each harmonic and then inserting the values in the calculation equation (1) or (2). In this way the major part of the noise, the hum and the remnants of the fundamental do not intervene anymore. In audio-frequency domain, this method can be applied in various ways:

- by using a selective voltmeter with manual tuning, to successively measure each signal component
- by using an sweeping analyzer
- by using analyzers with fixed set of filters (example the real time analyzers, RTA, used in acoustics having filters of 1/3 octave band or narrower)
- by using FFT analyzers (some having a function for distortion measurements).

We expect that measurement uncertainty is due to a) the errors in measuring the harmonics, b) to neglected harmonics in the upper frequency domain and, c) when the test is long, due to the stability of both measured signal and instrumentation. The noise affecting the test is only the one present in the narrow band where the fundamentals are measured.

The error in measuring the amplitudes comes from different causes. In a similar way as shown for the distortion meter and in Figure 1, the accuracy of the test instrument, which includes the linearity and the flatness of its frequency response, will be one of the causes. What is sometimes overlooked is the supplementary uncertainty arising from the tuning of the measuring filter on the frequency of the measured harmonic.

3.1 Analyzers with adjustable tuning

We discuss here the selective voltmeter and the swept analyzers. The supplementary error due to tuning is illustrated in Figure 2.

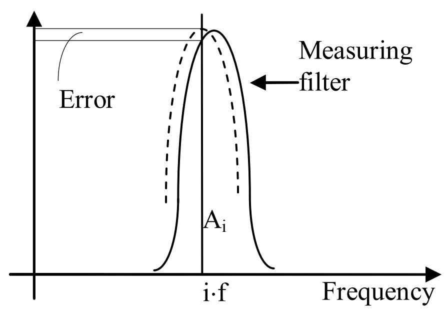

Figure 2. Amplitude error due to tuning

The filter response to the i^{th} harmonic, with frequency i times f_1 (with f_1 the frequency of the fundamental), depends on the tuning error and the filter shape, that is the flatness of the characteristic around the central frequency. The selectivity of the filter, which determines the resolution power of the analyzer, is an influencing factor. Most of the time, this is an adjustable parameter. By choosing a lower selectivity, remaining still in the domain of acceptable resolution, one may diminish the error due to tuning. The down side is that there will be more noise added on the signal, corresponding to the larger frequency band of the filter, but still very low compared to the distortion meter method.

In the sweeping analyzers, there is a supplementary source of error due to the sweeping speed. If the speed is too high, the filter does not have time to respond to the various signals. Consequently the measured levels are lower than in reality and the indicated frequency is shifted. This effect is accentuated if the filter selectivity is high (narrow band) and the frequency span is large. In general the instruction manual of the analyzer details the correct test conditions and frequently there are indicators for parameters mismatch on the front panel of the instruments.

With the digital filters, and even more with FFT instruments, because the calculations are done in parallel, there is an improvement in the measurement time, therefore the speed of analysis is superior and the results less influenced by signal instability.

3.2 Real time analyzers with fixed filters

A commonly used instrument in acoustics is the real time analyzer (RTA). The filters have a selectivity of a fraction of an octave and a fixed center frequency. The relative attenuation and central frequencies are established in standards. Often, the frequency domain of these instruments is limited to 20 kHz. When measuring a signal with these filters, the relative position of the signal and the filter central frequency will determine the accuracy of measurement of the level, as presented in Figure 3.

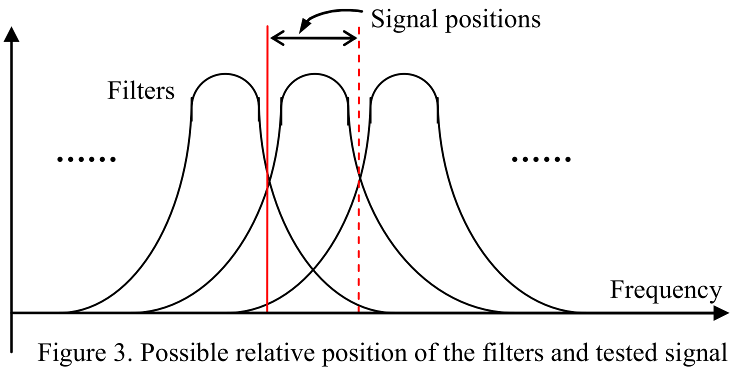

Figure 3. Possible relative position of the filters and tested signal frequency (equivalent to mistuning).

From the nominal characteristic, the filter attenuation for the possible position of the signal on the filter shape, can be anywhere between 0 dB and -3 dB (that is between 0 and –29 %). These values are affected by an additional ± 1.5 dB error due to the accuracy of the filter present in the analyzer used. The admissible tolerances are given by standards for different accuracy classes of filters.

In order to avoid this error, the method is therefore corrected: the amplitudes measured are calculated from the responses of multiple filters (k), which collect the energy of the signal.. These sums of squares of the filter responses replace each component amplitude A_i in the relations defining distortion as shown below.

In Equations (1) and (8) each of the harmonics of amplitude A_i, i = 1, 2, .., n (1 stands for the fundamental) is replaced with a series of k signals, where k is the number of the adjacent filters containing information about the measured component ($k \geq 3$). The squared RMS value of the harmonic A_i is:

$$A_i^2 = \sum_{j=1}^{k} A_{i,j}^2 \qquad (9)$$

The filters relative attenuations are specified in standards and are such that the resulting value of the A_i is not attenuated.

The uncertainties on the amplitude components participating to the final amplitude value of each harmonic are due to the filters' accuracies at different frequencies. To evaluate these is not an easy job, as tolerances vary with the frequency deviation from the center. Each of the amplitudes $A_{i,j}$ is affected by a relative uncertainty, $u_{i,j\,r}$. The resulting relative uncertainties $u_{i,r}$ on A_i is then:

$$u_{i,r}^2 = \frac{\sum_{j=1}^{k} A_{i,j}^2 u_{i,j,r}^2}{A_i^2} \qquad (10)$$

where A_i is defined by Equation (9).

The additional components with higher uncertainty are also the ones with the lowest level, because these come from filters that measure on the skirts of their characteristics. Consequently their influence is not a major one. The best case is when the signal frequency coincides with the filter center frequency, f_m. Then the harmonic amplitude is affected by the low uncertainty of the filter response in the middle of the range: u_{min} = 0.25 dB (or 3%). The other components added form the adjacent filters responses are too low and their uncertainty does not count in the budget. The worst case, as shown in Figure 3, is when the signal is in between filters: then two large components with high uncertainties (tolerances of 1.5 dB) are present in the sum, having a substantial influence. The standard uncertainty of the calculated amplitude of the harmonic based on two adjacent filter responses is u_{max} = 1 dB (or 12%).
Additional components for the resolution of the response reading for each filter must be taken into account.

Figure 4 below shows graphically the uncertainty evolution in function of the relative positions between the harmonics and the central frequency of the filter. The frequencies f_1 and f_2 correspond to the extreme signal positions shown in Figure 3.

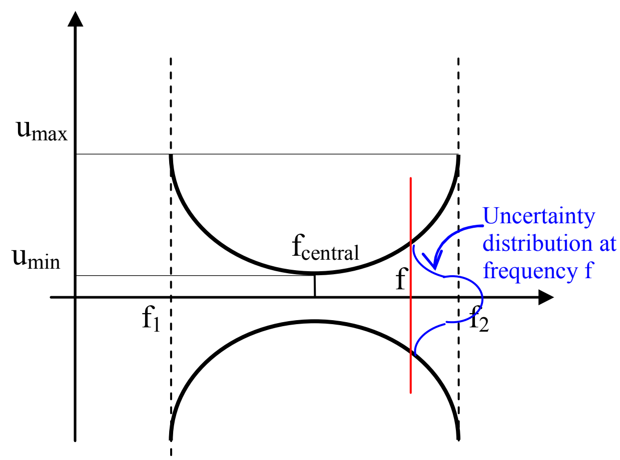

Figure 4. Evolution of the mistuning uncertainty along a 1/3 octave band filter

The exact shapes of the two curves depend on the specified filter tolerances at various frequencies.
In Figure 4, at each abscissa, the measurement error is considered Gaussian distributed. The corresponding measurement uncertainty for each harmonic amplitude is determined in function of its relative position from the filter central frequency. This is feasible when the frequency of the measured signal is known or predictable.

If we need to estimate the uncertainty for a signal that may have any frequency with equal probability, then this signal position with respect to the set of fixed 1/3 octave filters can be assumed to be in most cases within the interval:

$$f_{center} \pm \frac{f_{center} - f_1}{\sqrt{3}}, \tag{11}$$

as shown in Figure 5. We take as uncertainty on the measured amplitudes of the signal components the maximum value over this interval, which is u_f= 0.23 dB (or 2.6%).

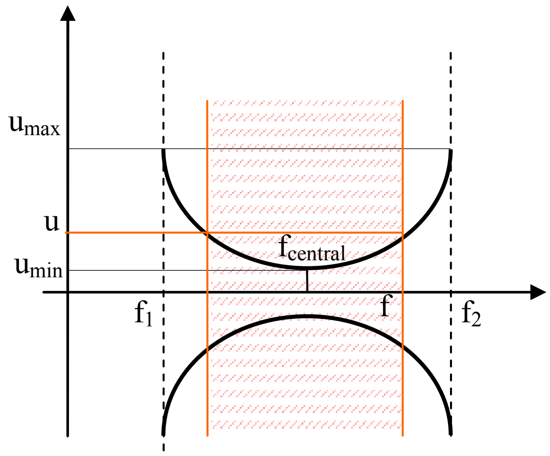

Figure 5. Most probable position of the signal components relative to the filters central frequency and the corresponding uncertainty on measured amplitude

Correlations

In order to determine if there is a correlation between the uncertainty components for the amplitude of the various harmonics, we need to establish if there is a correlation between the positions of the harmonics relative to the centers of the filters.

The 1/3 octave filters center frequencies, $f_{m,}$ are in a succession determined by the following equations (x = filter number):

$$f_m = 10^{\frac{x}{10}} \cdot 1000 \text{ for base ten filters, or} \tag{12}$$

$$f_m = 2^{(x-30)/3} \cdot 1000 \text{ for filters in base 2} \tag{13}$$

The ratio, R, between two successive center frequencies, in the $1/3^{rd}$ octave filters, is approximately 1.26 in both cases. As the frequency of the harmonics present in the signal are multiple of f_1, the frequency of the fundamental, then the pattern of the relative positions will repeat only after $R \cdot f_c$ times the frequency of the signal (or the lowest common multiple of $(R \cdot f_c)$ and f_1) which may be outside the measuring domain. Therefore, we conclude that the uncertainties on the harmonic amplitudes are not correlated.

The RTA has the advantage of averaging the measured values over a time interval, which may eliminate some of the superposed noise.

3.3 Fast Fourier Transform (FFT) analyzers

Due to the temporal limitation of the acquired signal sample, the signal energy is spread in a frequency domain. As in the previous case, the amplitude must be calculated from the response existing in several adjacent frequency lines (bins).

Unlike the previous case, where the amplitude attenuation is predictable if the relative position of the signal and filters central frequency is known, here the amplitude level, therefore the associated uncertainty, depends on the test conditions (analyzer settings):

- sampling rate,
- frequency span,
- frequency resolution,
- window used.

In addition, the analyzer specifications are not usable for evaluating this uncertainty. In most of the cases there is no statement of accuracy, not even for particular test conditions. Consequently, by not being able to evaluate the measurement uncertainties, it is very difficult to achieve traceability on the measurements performed with these analyzers.

Some FFT analyzers have a viewer harmonic cursor that will automatically show and measure the harmonics of a signal with frequency chosen with the regular cursor. This works only if there is coincidence between the frequency lines of the analyzer and the signal frequency. If there is a deviation between the two, then the additional cursors, appearing at multiples of the frequency of the main cursor, will not coincide with the signal harmonics anymore. (see Figure 4.) This is because the main cursor can be adjusted in steps determined by the frequency lines, and it cannot catch the real signal frequency.

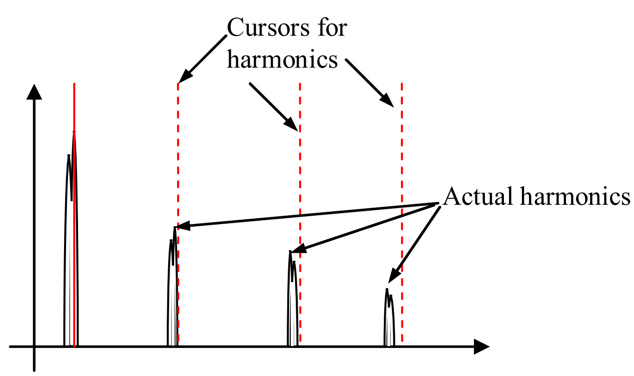

Figure 4. Erroneous measurements with FFT Analyzer using "show harmonics" function.

Consequently, the values read at those frequencies are not the amplitudes of the harmonics, but some lower values (noise or response of close lines activated by energy leakage.) We will not discuss the uncertainties in this case. If the analyzer does not have the possibility to fit the harmonics between the frequency lines, then this function, present in some analyzers, should not be used to measure signals that have frequencies uncorrelated with the analyzer's lines.

4 Comparison chart

After detailing the uncertainty sources affecting each of the most used methods for measuring the signal frequencies, it is time to compare these methods and determine if there is one that is ore accurate. Table 1 below summarizes the main uncertainty components for the methods and instruments discussed.

Table 1 Influence of error sources on various methods

Error source	Components	Distortion meter method	Selective voltmeter	RTA with 1/3 Octave band	FFT analyzer
Amplitude of the fundamental	Measurement accuracy (if ideally tuned)	Yes	Yes	Yes	unknown
	Analyzer tuning	Yes	Yes	No tuning	No tuning
	Additional components	No	No	Yes	Yes
	Rejection	Yes	n/a	n/a	n/a
	Stability	Yes	Yes: longest test	n/a	n/a
Amplitude of harmonics	Measurement accuracy (if ideally tuned)	Yes	Yes (low error)	Yes	Unknown
	Analyzer tuning	Yes	Yes	No tuning	No tuning
	Additional components	No	No	Yes	Yes
	Stability	Yes	Yes: longest test	n/a (simultaneous measurement)	n/a (simultaneous measurement)
Spurious signals	Noise, hum	Yes (may be major error source)	Negligible (Small noise in frequency band)	Negligible (Small noise in frequency band)	Negligible (Small noise in frequency band)

We give now an example from our practice: the distortions of the sound produced by an acoustical calibrator. The specifications for the tested device are:

- frequency 1000 Hz ± 1%,
- level 114 dB ± 0.4 dB,
- distortion less than 3%, and
- short term level fluctuation less than 0.1 dB (or 1.1%).

The sound is measured using a microphone and a microphone preamplifier, which are successively connected to the test instruments that use the methods detailed here.
The frequency characteristic of the microphone and preamplifier will influence the measurements in the same way in all cases. The same is for the other common error source: the limitations in the frequency domain do not allow the measurement of the harmonics of superior order.

Using the shown methods, we obtained the distortion values and estimated uncertainties as shown in Figure 5.

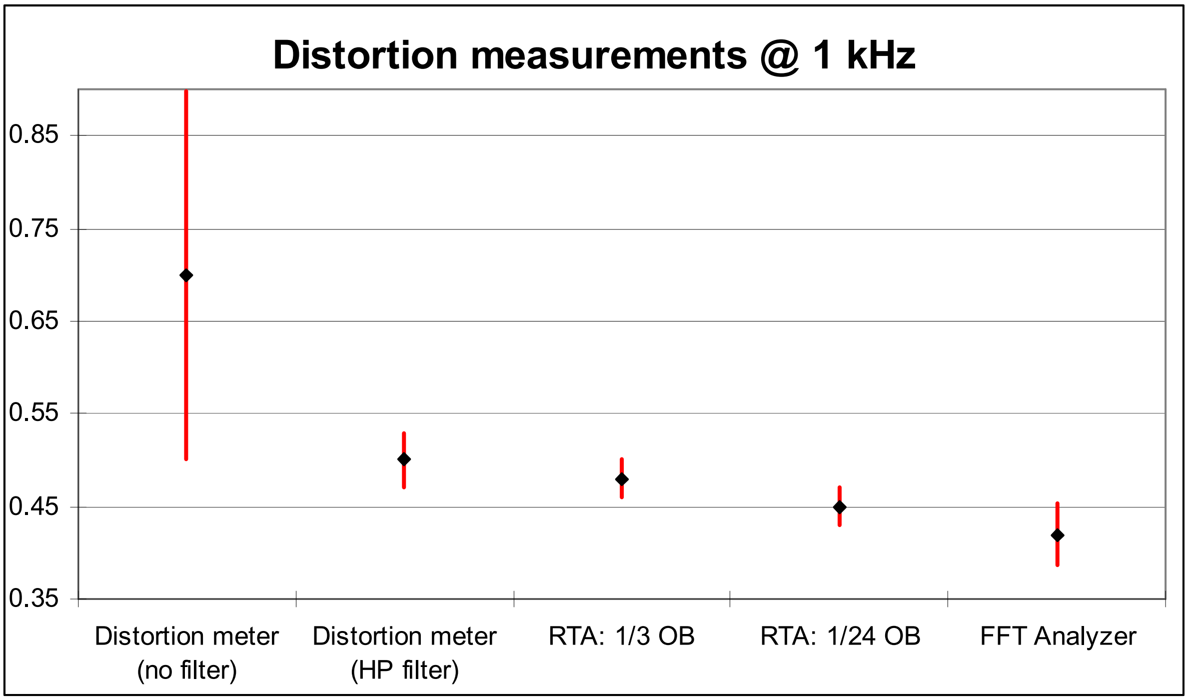

Figure 5. Signal distortion as measured with various instruments

As expected, the measurement uncertainties for the methods used were comparable, when special measures were taken to minimize the signal-to-noise ratio in the distortion meter method.

5 Conclusion

While analyzing the uncertainty sources in some common methods for distortion measurement methods, we pointed on uncertainty sources that may be unacceptable high if some precautions are not taken. Also, it is shown that more modern instruments, like FFT analyzers, may raise a traceability issue due to difficulties in evaluating the measurements uncertainties.
From all methods presented herein, we would choose to use the selective voltmeter with manual tuning or the super-heterodyne analyzer, as accurate, and insensitive to noise and spurious signals.

References

1. ANSI S1.11 2004 Specification for octave band and fractional octave band analog and digital filters.
2. IEC 1260 1995 Electroacoustics: Octave band and fractional octave band filters.
3. ISO/IEC/OIML Guide to the expression of the uncertainty in measurement.